Name ______________________ Class ______________ Date ____________

Skills Worksheet

Problem Solving

Gas Laws

Chemists found that there were relationships among temperature, volume, pressure, and quantity of a gas that could be described mathematically. This chapter deals with Boyle's law, Charles's law, Gay-Lussac's law, the combined gas law, and Dalton's law of partial pressures. These laws have one condition in common. They all assume that the molar amount of gas does not change. In other words, these laws work correctly only when no additional gas enters a system and when no gas leaks out of it. Remember also that a law describes a fact of nature. Gases do not "obey" laws. The law does not dictate the behavior of the gas. Rather, each gas law describes a certain behavior of gas that occurs if conditions are right.

BOYLE'S LAW

Robert Boyle, a British chemist who lived from 1627 to 1691 formulated the first gas law, now known as Boyle's law. This law describes the relationship between the pressure and volume of a sample of gas confined in a container. Boyle found that gases compress, much like a spring, when the pressure on the gas is increased. He also found that they "spring back" when the pressure is lowered. By "springing back" he meant that the volume increases when pressure is lowered. It's important to note that Boyle's law is true only if the temperature of the gas does not change and no additional gas is added to the container or leaks out of the container.

Boyle's law states that *the volume and pressure of a sample of gas are* <u>*inversely*</u> *proportional to each other at constant temperature.* This statement can be expressed as follows.

Proportionality symbol. It means "is proportional to."

Proportionality constant

$$V \propto \frac{1}{P} \quad \text{and} \quad PV = k \quad \text{or} \quad V = k\frac{1}{P}$$

According to Boyle's law, when the pressure on a gas is *increased,* the volume of the gas *decreases.* For example, if the pressure is doubled, the volume decreases by half. If the volume quadruples, the pressure decreases to one-fourth of its original value.

The expression $PV = k$ means that the product of the pressure and volume of any sample of gas is a constant, k. If this is true, then $P \times V$ under one set of conditions is equal to $P \times V$ for the same sample of gas under a second set of conditions, as long as the temperature remains constant.

Boyle's law can be expressed by the following mathematical equation.

$$P_1V_1 = P_2V_2$$

Pressure under the *first* set of conditions × Volume under the *first* set of conditions = Pressure under the *second* set of conditions × Volume under the *second* set of conditions

General Plan for Solving Boyle's-Law Problems

1 Given three of the following four quantities: P_1, V_1, P_2, V_2

↓ Rearrange the equation $P_1V_1 = P_2V_2$ algebraically to solve for the unknown quantity.

2 An equation that can be used to calculate the unknown quantity
It will be one of the following four:

$$V_2 = \frac{P_1V_1}{P_2},\ P_2 = \frac{P_1V_1}{V_2},\ V_1 = \frac{P_2V_2}{P_1},\ P_1 = \frac{P_2V_2}{V_1}$$

↓ Substitute each of the known quantities, and calculate.

3 Unknown P or V

Sample Problem 1

A sample of nitrogen collected in the laboratory occupies a volume of 725 mL at a pressure of 0.971 atm. What volume will the gas occupy at a pressure of 1.40 atm, assuming the temperature remains constant?

Solution

ANALYZE

What is given in the problem? **the original volume and pressure of the nitrogen sample, and the new pressure of the sample**

What are you asked to find? **the volume at the new pressure**

Items	**Data**
Original pressure, P_1	0.971 atm
Original pressure, V_1	725 mL N_2
New pressure, P_2	1.40 atm
New volume, V_2	? mL N_2

PLAN

What steps are needed to calculate the new volume of the gas?

Rearrange the Boyle's law equation to solve for V_2, substitute known quantities, and calculate.

$$\boxed{1}\quad P_1V_1 = P_2V_2 \xrightarrow[\textit{insert data and solve for } V_2]{\textit{to solve for } V_2\textit{, divide both sides of the equation by } P_2 \textit{ to isolate } V_2} \boxed{2}\quad V_2 = \frac{P_1V_1}{P_2}$$

COMPUTE

Substitute data for the terms of the equation, and compute the result.

$$V_2 = \frac{P_1V_1}{P_2} = \frac{0.971 \cancel{\text{atm}} \times 725 \text{ mL } N_2}{1.40 \cancel{\text{atm}}} = 503 \text{ mL } N_2$$

EVALUATE

Are the units correct?

Yes; units canceled to give mL N_2.

Is the number of significant figures correct?

Yes; the number of significant figures is correct because data were given to three significant figures.

Is the answer reasonable?

Yes; pressure increased by about 1/3, volume must decrease by about 1/3.

Charles's law states that *the volume of a sample of gas is directly proportional to the absolute temperature when pressure remains constant.* Charles's law can be expressed as follows.

$$V \propto T \qquad \text{and} \qquad \frac{V}{T} = k, \qquad \text{or} \qquad V = kT$$

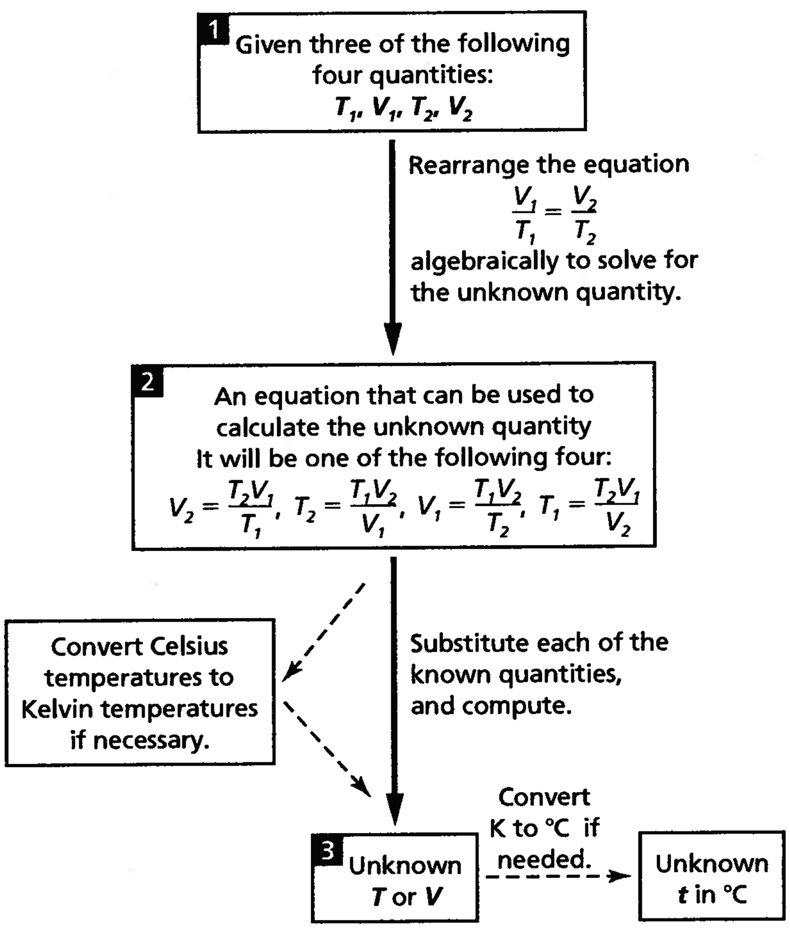
According to Charles's law, when the temperature of a sample of gas *increases*, the volume of the gas *increases* by the same factor. Therefore, doubling the Kelvin temperature of a gas will double its volume. Reducing the Kelvin temperature by 25% will reduce the volume by 25%.

The expression $V/T = k$ means that the result of volume divided by temperature is a constant, k, for any sample of gas. If this is true, then V/T under one set of conditions is equal to V/T for the same sample of gas under another set of conditions, as long as the pressure remains constant.

Charles's law can be expressed by the following mathematical equation.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

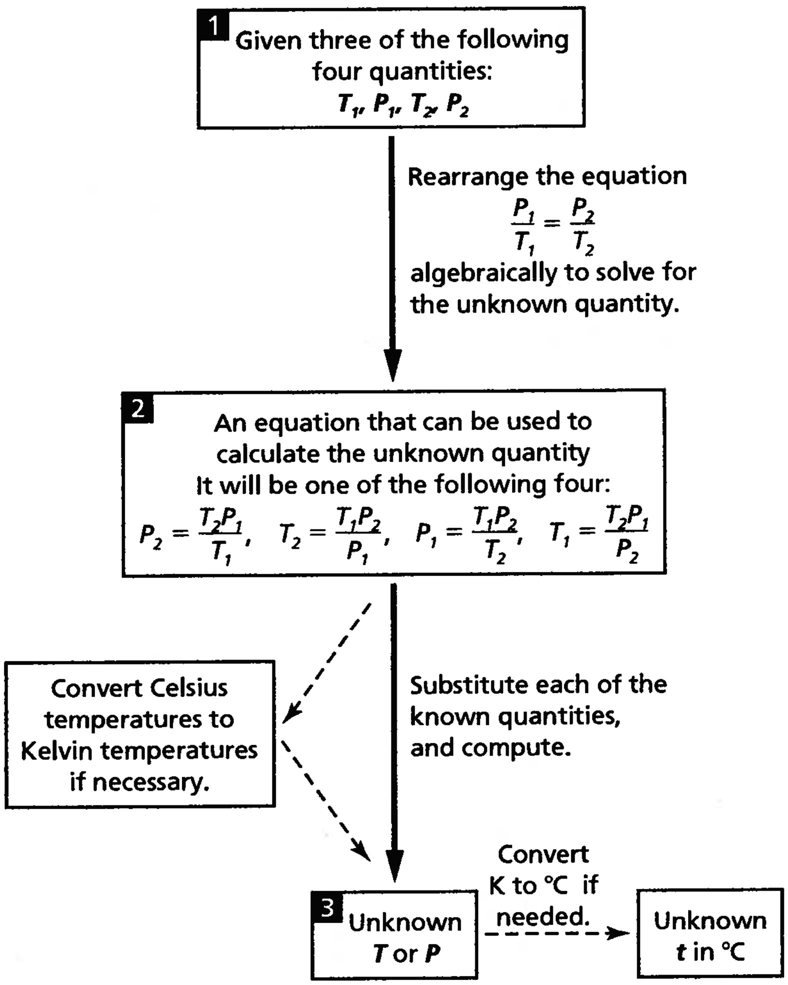
General Plan for Solving Charles's-Law Problems



Gay-Lussac's law can be expressed by the following mathematical equation.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

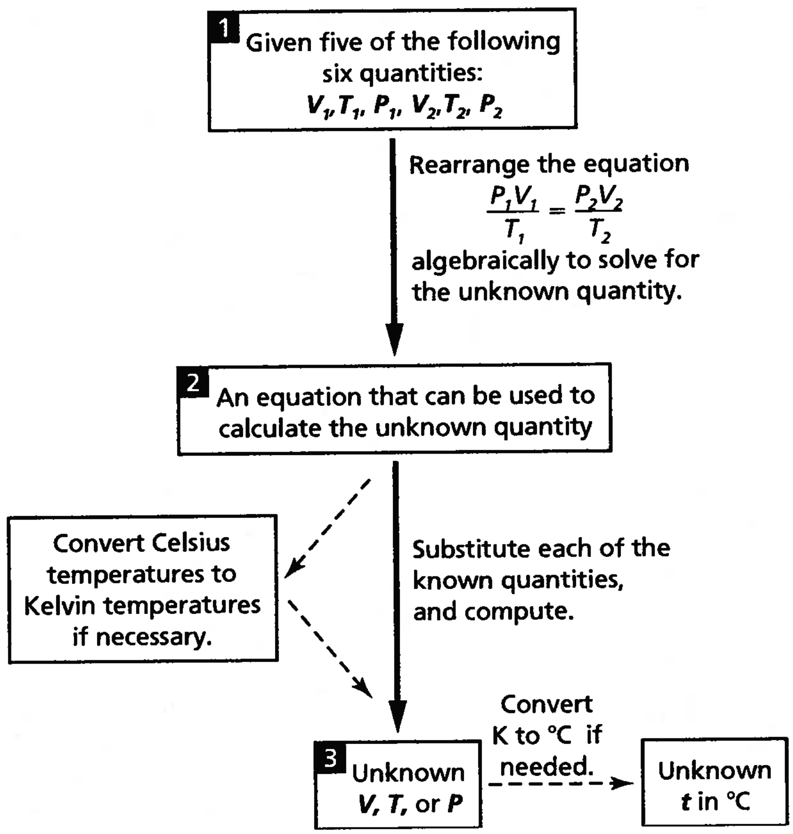
General Plan for Solving Gay-Lussac's-Law Problems



volume, and temperature of a gas vary. Only the molar quantity of the gas must be constant.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

General Plan for Solving Combined-Gas-Law Problems



DALTON'S LAW OF PARTIAL PRESSURES

Air is a mixture of approximately 78% N_2, 20% O_2, 1% Ar, and 1% other gases by volume, so at any barometric pressure 78% of that pressure is exerted by nitrogen, 20% by oxygen, and so on. This phenomenon is described by Dalton's law of partial pressures, which says that *the total pressure of a mixture of gases is equal to the sum of the partial pressures of the component gases.* It can be stated mathematically as follows.

$$P_{Total} = P_{Gas\ 1} + P_{Gas\ 2} + P_{Gas\ 3} + P_{Gas\ 4} + \ldots$$

A common method of collecting gas samples in the laboratory is to bubble the gas into a bottle filled with water and allow it to displace the water. When this technique is used, however, the gas collected in the bottle contains a small but significant amount of water vapor. As a result, the pressure of the gas that has displaced the liquid water is the sum of the pressure of the gas plus the vapor pressure of water at that temperature. The vapor pressures of water at various temperatures are given in **Table 1** below.

TABLE 1 VAPOR PRESSURE OF WATER

Temp in °C	Vapor pressure in kPa	Temp in °C	Vapor pressure in kPa	Temp in °C	Vapor pressure in kPa
10	1.23	17	1.94	24	2.98
11	1.31	18	2.06	25	3.17
12	1.40	19	2.19	26	3.36
13	1.50	20	2.34	27	3.57
14	1.60	21	2.49	28	3.78
15	1.71	22	2.64	29	4.01
16	1.82	23	2.81	30	4.25

$$P_{Total} = P_{Gas} + P_{H_2O\ vapor}$$

To find the true pressure of the gas alone, the pressure of the water vapor must be subtracted from the total pressure.

$$P_{Gas} = P_{Total} - P_{H_2O\ vapor}$$

You can use this corrected pressure in gas-law calculations to determine what the volume of the gas alone would be.

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Skills Worksheet

Problem Solving

The Ideal Gas Law

In 1811, the Italian chemist Amedeo Avogadro proposed the principle that *equal volumes of gases at the same temperature and pressure contain equal numbers of molecules.* He determined that at standard temperature and pressure, one mole of gas occupies 22.414 10 L (usually rounded to 22.4 L).

At this point, if you know the number of moles of a gas, you can use the molar volume of 22.4 L/mol to calculate the volume that amount of gas would occupy at STP. Then you could use the combined gas law to determine the volume of the gas under any other set of conditions. However, a much simpler way to accomplish the same task is by using the ideal gas law.

The *ideal gas law* is a mathematical relationship that has the conditions of standard temperature (273 K) and pressure (1 atm or 101.3 kPa) plus the molar gas volume (22.4 L/mol) already combined into a single constant. The following equation is the mathematical statement of the ideal gas law.

$$PV = nRT$$

in which

P = the pressure of a sample of gas

V = the volume of a sample of gas

n = the number of moles of gas present

T = the Kelvin temperature of the gas

R = the ideal gas constant, which combines standard conditions and molar volume into a single constant

The value of the ideal gas constant, R, depends on the units of P and V being used in the equation. Temperature is always in kelvins and amount of gas is always in moles. The most common values used for R are shown below.

Units of *P* and *V*	Value of *R*
Atmospheres and liters	$0.0821 \frac{L \cdot atm}{mol \cdot K}$
Kilopascals and liters	$8.314 \frac{L \cdot kPa}{mol \cdot K}$

If you have volume units other than liters or pressure units other than atmospheres or kilopascals, it is best to convert volume to liters and pressure to atmospheres or kilopascals.

General Plan for Solving Ideal-Gas-Law Problems

1 The equation for the ideal gas law
$PV = nRT$

↓ Determine from the data which is the unknown quantity. Rearrange the equation algebraically to solve for the unknown quantity.

2 An equation that can be used to calculate the unknown quantity

↓ Choose the gas constant, *R*, that best fits the units of the data. Substitute each of the data values in the equation and calculate.

3 Unknown
P, V, n, or ***T***

Sample Problem 1

An engineer pumps 5.00 mol of carbon monoxide gas into a cylinder that has a capacity of 20.0 L. What is the pressure in kPa of CO inside the cylinder at 25°C?

Solution

ANALYZE

What is given in the problem? **the amount in moles of gas pumped into the cylinder, the volume of the cylinder, and the temperature**

What are you asked to find? **the pressure of the gas in the cylinder**

Items	Data
Amount of gas, n	5.00 mol
Volume of gas in cylinder, V	20.0 L
Temperature of gas, t	25°C
Kelvin temperature of gas, T	(25 + 273) K = 298 K
Ideal gas constant, R	0.0821 L·atm/mol·K or 8.314 L·kPa/mol·K
Pressure in cylinder, P	? kPa

PLAN

What steps are needed to calculate the new pressure of the gas?

Rearrange the ideal-gas-law equation to solve for *P*, substitute known quantities, and calculate.

1
Ideal-gas-law equation,
$PV = nRT$

↓ *solve the ideal-gas-law equation for pressure*

2
$$P = \frac{nRT}{V}$$

↓ *the problem asks for answer in kPa, so choose the appropriate R, substitute known values, and solve*

3
Unknown pressure,
P

$$PV = nRT$$

Solve the ideal-gas-law equation for P, the unknown quantity.

$$P = \frac{nRT}{V}$$

COMPUTE

The problem asks for pressure in kPa, so use $R = 8.314$ L·kPa/mol·K.

$$P = \frac{5.00 \cancel{\text{mol}} \times 8.314 \cancel{\text{L}}\cdot\text{kPa}/\cancel{\text{mol}\cdot\text{K}} \times 298 \cancel{\text{K}}}{20.0 \cancel{\text{L}}} = 619 \text{ kPa}$$

EVALUATE

Are the units correct?

Yes; the ideal gas constant was selected so that the units canceled to give kPa.

Is the number of significant figures correct?

Yes; the number of significant figures is correct because data were given to three significant figures.

Is the answer reasonable?

Yes; the calculation can be approximated as (1/4) × (8 × 300), or 2400/4, which equals 600. Thus, 619 kPa is in the right range.

Practice

1. A student collects 425 mL of oxygen at a temperature of 24°C and a pressure of 0.899 atm. How many moles of oxygen did the student collect?
   ans: 1.57×10^{-2} mol O_2

APPLICATIONS OF THE IDEAL GAS LAW

You have seen that you can use the ideal gas law to calculate the moles of gas, n, in a sample when you know the pressure, volume, and temperature of the sample. When you know the amount and identity of the substance, you can use its molar mass to calculate its mass. You did this when you learned how to convert between mass and moles. The relationship is expressed as follows.

Mass in grams

$$n = \frac{m}{M}$$

Molar mass in grams per mole

Amount in moles

If you substitute the expression m/M for n in the ideal-gas-law equation, you get the following equation.

$$PV = \frac{m}{M}RT$$

This version of the ideal gas law can be solved for any of the five variables P, V, m, M, or T. It is especially useful in determining the molecular mass of a substance. This equation can also be related to the density of a gas. Density is mass per unit volume, as shown in the following equation.

$$D = \frac{m}{V}$$

Solve for m:

$$m = DV$$

Then, substitute DV for m in the gas law equation:

$$PV = \frac{DV}{M}RT$$

The two V terms cancel and the equation is rearranged to give:

$$PM = DRT \qquad \text{or} \qquad D = \frac{PM}{RT}$$

This equation can be used to compute the density of a gas under any conditions of temperature and pressure. It can also be used to calculate the molar mass of an unknown gas if its density is known.

General Plan for Solving Problems Involving Applications of the Ideal Gas Law

1a $PV = \frac{m}{M}RT$

Determine which equation fits the problem.

1b $D = \frac{PM}{RT}$

Rearrange the equation algebraically to solve for the unknown quantity.

2 An equation that can be used to calculate the unknown quantity

Choose the gas constant, *R*, that best fits the units of the data. Substitute each of the data values in the equation and calculate.

3 Unknown
P, V, m, M, D or ***T***

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Skills Worksheet

Problem Solving

Stoichiometry of Gases

Now that you have worked with relationships among moles, mass, and volumes of gases, you can easily put these to work in stoichiometry calculations. Many reactions have gaseous reactants, gaseous products, or both.

Reactants and products that are not gases are usually measured in grams or kilograms. As you know, you must convert these masses to amounts in moles before you can relate the quantities by using a balanced chemical equation. Gaseous products and reactants can be related to solid or liquid products and reactants by using the mole ratio, just as solids and liquids are related to each other.

Reactants and products that are gases are usually measured in liters. If the gas is measured at STP, you will need only Avogadro's law to relate the volume and amount of a gas. One mole of any gas at STP occupies 22.4 L. If the gas is not at STP, you will need to use the ideal gas law to determine the number of moles. Once volume has been converted to amount in moles you can use the mole ratios of products and reactants to solve stoichiometry problems involving multiple phases of products and reactants.

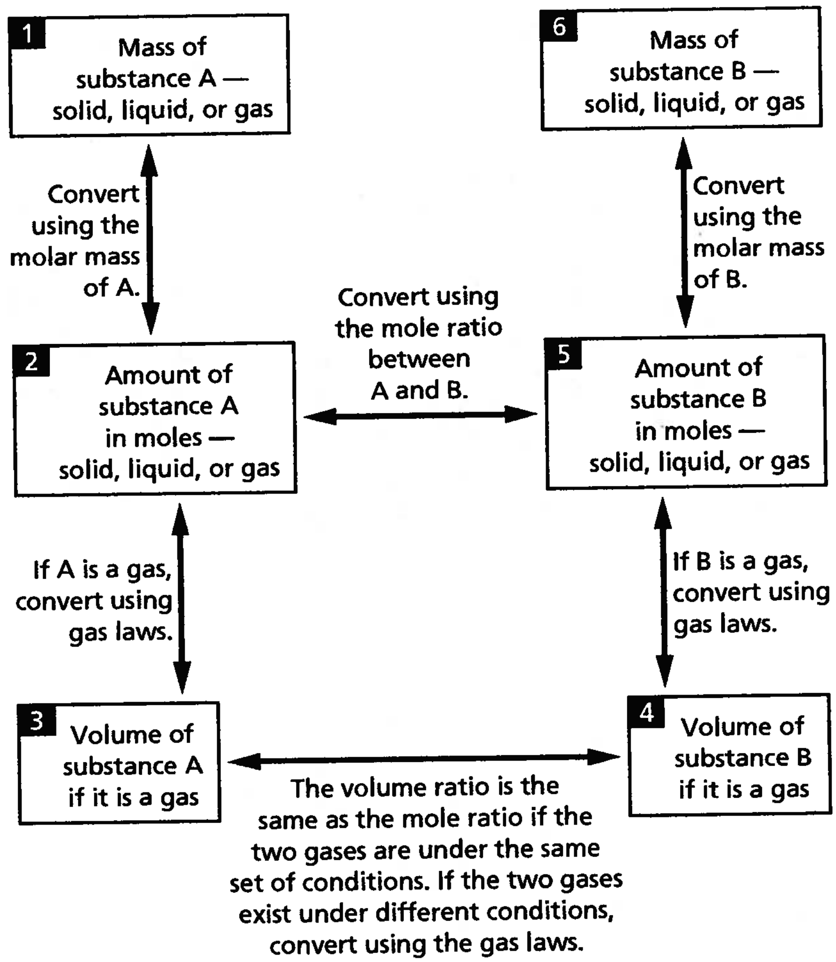
$$n = \frac{PV}{RT}$$

If the problem which you are trying to solve involves only gases, there is a simpler way of dealing with the stoichiometric amounts. Look again at the expression for the ideal gas law above; the molar amount of a gas is directly related to its volume. Therefore, the mole ratios of gases given by the coefficients in the balanced equation can be used as volume ratios of those gases to solve stoichiometry problems. No conversion from volume to amount is required to determine the volume of one gas from the volume of another gas in a balanced chemical equation.

There is one condition that must be observed. Gas volumes can be related by mole ratios only when the volumes are measured under the same conditions of temperature and pressure. If they are not, then the volume of one of the gases must be converted to the conditions of the other gas. Usually you will need to use the combined gas law for this conversion.

$$V_2 = \frac{V_1 P_1 T_2}{T_1 P_2}$$

General Plan for Solving Gas Stoichiometry Problems



Sample Problem 1

Ammonia can react with oxygen to produce nitrogen and water according to the following equation.

$$4NH_3(g) + 3O_2(g) \rightarrow 2N_2(g) + 6H_2O(l)$$

If 1.78 L of O_2 reacts, what volume of nitrogen will be produced? Assume that temperature and pressure remain constant.

Solution

ANALYZE

What is given in the problem? **the balanced equation, the volume of oxygen, and the fact that the two gases exist under the same conditions**

What are you asked to find? **the volume of N_2 produced**

Items	**Data**	
Substance	O_2	N_2
Coefficient in balanced equation	3	2
Molar mass	NA	NA
Moles	NA	NA
Mass of substance	NA	NA
Volume of substance	1.78 L	? L
Temperature conditions	NA	NA
Pressure conditions	NA	NA

PLAN

What steps are needed to calculate the volume of N_2 formed from a given volume of O_2?

The coefficients of the balanced equation indicate the mole ratio of O_2 to N_2. The volume ratio is the same as the mole ratio when volumes are measured under the same conditions.

$$\underset{\text{in L}}{\boxed{3}\ \text{Volume of } O_2} \xrightarrow[\textit{multiply by the volume ratio, } \frac{N_2}{O_2}]{} \underset{\text{in L}}{\boxed{4}\ \text{Volume of } N_2}$$

$$\overset{given}{L\,O_2} \times \overset{\textit{volume ratio, } \frac{N_2}{O_2}}{\frac{2\,L\,N_2}{3\,L\,O_2}} = L\,N_2$$

COMPUTE

$$1.78\ \cancel{L\,O_2} \times \frac{2\ L\ N_2}{3\ \cancel{L\,O_2}} = 1.19\ L\ N_2$$

EVALUATE

Are the units correct?

Yes; units canceled to give L N_2.

Is the number of significant figures correct?

Yes; the number of significant figures is correct because the data were given to three significant figures.

Is the answer reasonable?

Yes; the volume of N_2 should be 2/3 the volume of O_2.

Practice

1. In one method of manufacturing nitric acid, ammonia is oxidized to nitrogen monoxide and water.

$$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(l)$$

What volume of oxygen will be used in a reaction of 2800 L of NH_3? What volume of NO will be produced? All volumes are measured under the same conditions. **ans: 3500 L O_2, 2800 L NO**

2. Fluorine gas reacts violently with water to produce hydrogen fluoride and ozone according to the following equation.

$$3F_2(g) + 3H_2O(l) \rightarrow 6HF(g) + O_3(g)$$

What volumes of O_3 and HF gas would be produced by the complete reaction of 3.60×10^4 mL of fluorine gas? All gases are measured under the same conditions. **ans: 1.20×10^4 mL O_3, 7.20×10^4 mL HF**